Q.P Code: 71865

KINEMATICS OF MACHINERY SOLUTION

SEM 4 (CBCGS-MAY 2019)

BRANCH-MECHANICAL ENGINEERING

Q 1) A) State and prove kennedy's Theorem. (05)

Solution:

The Kennedy's theorem states that if three bodies move relatively to each other, they have three instantaneous centres and lie on a straight line.

Consider three kinematic links *A*, *B* and *C* having relative plane motion. The number of instantaneous centres (*N*) is given by

$$N = \frac{n(n-1)}{2} = \frac{3(3-1)}{2} = 3$$

Where $n = \text{Number of links} = 3$

The two instantaneous centres at the pin joints of *B* with *A*, and *C* with *A* (*i.e.* I_{ab} and I_{ac}) are the permanent instantaneous centres. According to Aronhold Kennedy's theorem, the third instantaneous centre I_{bc} must lie on the line joining I_{ab} and I_{ac}. In order to prove this, let us consider that the instantaneous centre *Ibc* lies outside the line joining I_{ab} and I_{ac} as shown in Fig. The point I_{bc}belongs to both the links *B* and *C*. Let us consider the point I_{bc} on the link *B*. Its velocity v_{BC} must be perpendicular to the line joining $I_{ab}$and I_{bc}. Now consider the point I_{bc} on the link *C*. Its velocity v_{BC} must be perpendicular to the line joining $I_{ac}$and I_{bc}.

We have already discussed in Art, that the velocity of the instantaneous centre is same whether it is regarded as a point on the first link or as a point on the second link. Therefore, the velocity of the point I_{bc} cannot be perpendicular to both lines I_{ab} I_{bc} and $I_{ac}I_{bc}$unless the point I_{bc} lies on the line joining the points I_{ab} and I_{ac}. Thus the three instantaneous centres (I_{ab}, I_{ac} and I_{bc}) must lie on the same straight line. The exact location of I_{bc} on line I_{ab} I_{ac} depends upon the directions and magnitudes of the angular velocities of *B* and *C* relative to *A*.

Q 1) B) Define i) Kinematic link ii) kinematic pair iii) Kinematic chain (05)

Solution:

i) Kinematic link :

Each part of a machine, which moves relative to some other part, is known as a kinematic link (or simply link) or element.

ii) Kinematic Pair :

The two links or elements of a machine, when in contact with each other, are said to form a pair. If the relative motion between them is completely or successfully constrained (i.e. in a definite direction), the pair is known as kinematic pair.

iii) Kinematic Chain :

When the kinematic pairs are coupled in such a way that the last link is joined to the first link to transmit definite motion (i.e. completely or successfully constrained motion), it is called a kinematic chain. In other words, a kinematic chain may be defined as a combination of kinematic pairs, joined in such a way that each link forms a part of two pairs and the relative motion between the links or elements is completely or successfully constrained.

Q 1) C) Classify follower in details. (05)

Solution:

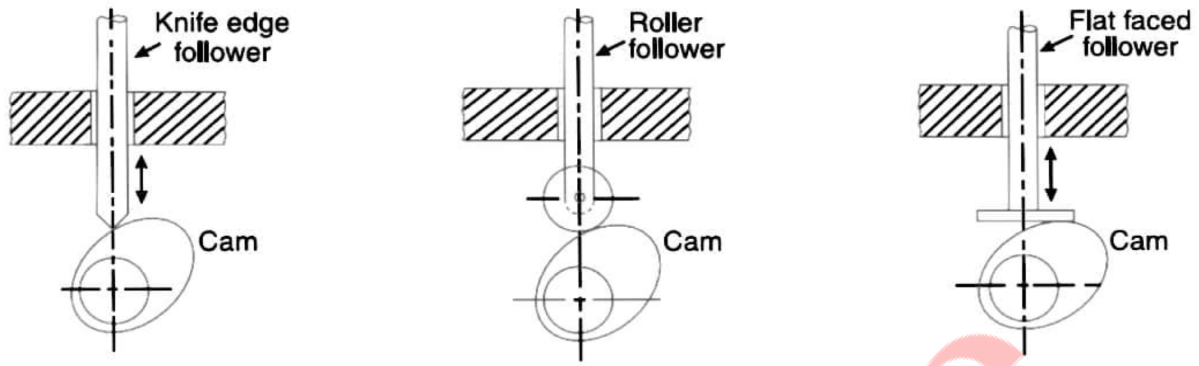
The followers may be classified as discussed below :

1. According to the surface in contact. The followers, according to the surface in contact, are as follows:

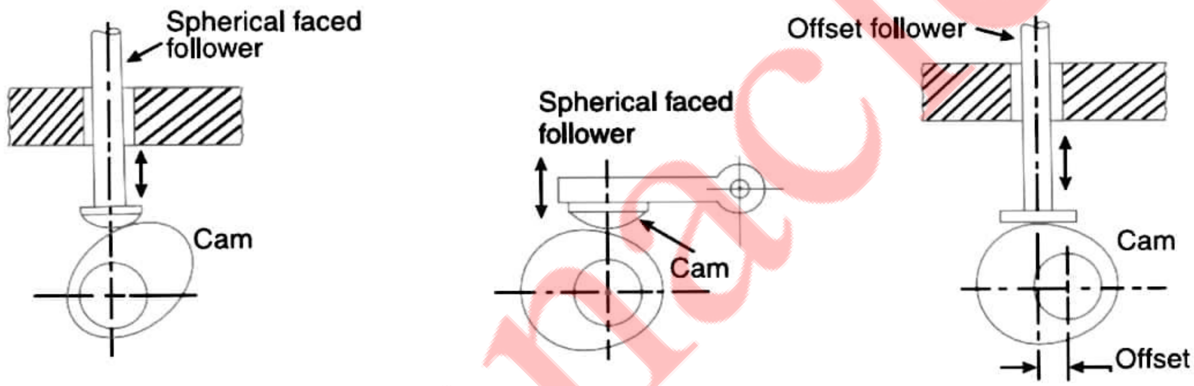
(a) Knife edge follower: When the contacting end of the follower has a sharp knife edge, it is called a knife edge follower, as shown in Fig. (a). The sliding motion takes place between the contacting surfaces (i.e. the knife edge and the cam surface). It is seldom used in practice because the small area of contacting surface results in excessive wear. In knife edge followers, a considerable side thrust exists between the follower and the guide.

(b) Roller follower: When the contacting end of the follower is a roller, it is called a roller follower, as shown in Fig. (b). Since the rolling motion takes place between the contacting surfaces (i.e. the roller and the cam), therefore the rate of wear is greatly reduced. In roller followers also the side thrust exists between the follower and the guide. The roller followers are extensively used where more space is available such as in stationary gas and oil engines and aircraft engines.

(c) Flat faced or mushroom follower: When the contacting end of the follower is a perfectly flat face, it is called a flat-faced follower, as shown in Fig. (c). It may be noted that the side thrust between the follower and the guide is much reduced in case of flat faced followers. The only side thrust is due to friction between the contact surfaces of the follower and the cam. The relative motion between these surfaces is largely of sliding nature but wear may be reduced by off-setting the axis of the follower, as shown in Fig. (f) so that when the cam rotates, the follower also rotates about its own axis. The flat faced followers are generally used where space is limited such as in cams which operate the valves of automobile engines.



(*a*) Cam with knife edge follower. (*b*) Cam with roller follower. (*c*) Cam with flat faced follower.



(*d*) Cam with spherical faced follower. (*e*) Cam with spherical faced follower. (*f*) Cam with offset follower.

(d) Spherical faced follower: When the contacting end of the follower is of spherical shape, it is called a spherical faced follower, as shown in Fig.(d). It may be noted that when a flat-faced follower is used in automobile engines, high surface stresses are produced. In order to minimize these stresses, the flat end of the follower is machined to a spherical shape.

2. According to the motion of the follower. The followers, according to its motion, are of the following two types:

(a) Reciprocating or translating follower: When the follower reciprocates in guides as the cam rotates uniformly, it is known as reciprocating or translating follower. The followers as shown in Fig. (a) to (d) are all reciprocating or translating followers.

(b) Oscillating or rotating follower: When the uniform rotary motion of the cam is converted into predetermined oscillatory motion of the follower, it is called oscillating or rotating follower. The follower, as shown in Fig (e), is an oscillating or rotating follower.

3. According to the path of motion of the follower. The followers, according to its path of motion, are of the following two types:

(a) Radial follower. When the motion of the follower is along an axis passing through the centre of the cam, it is known as radial follower. The followers, as shown in Fig. (a) to (e), are all radial followers.

(b) Off-set follower: When the motion of the follower is along an axis away from the axis of the cam centre, it is called off-set follower. The follower, as shown in Fig. (f), is an off-set follower.

Q 1) D) Explain self energizing and self locking brake. (05)

Solution:

i) Self energizing brake:

1) The frictional force helps to apply the brake. Such type of brakes are said to be self energizing brakes.

2) $P = \frac{R_n(x-\mu.a)}{l}$ $\quad\quad$ $R_n x - \mu.R_n.a = P.l$

$R_n.x = P.l + \mu.R_n.a$

From the above equation we can say that moment of frictional force $(\mu.R_n.a)$ adds to the moment force $(p.l)$. In other word frictional force helps to apply brake. Such types of brakes are called self energizing brakes.

ii) Self locking brakes :

1) When the frictional force is great enough to apply the brake with no external force, then the brake is said to be self-locking brake

2) $R_n = \frac{P.l}{x-\mu.a}$ $\quad\quad$ $P = \frac{R_n(x-\mu.a)}{l}$

In above equation if $x \leq \mu.a$, the effort P become negative or zero. Thus no effort is required to apply brakes and hence brake is self locking.

Therefore, condition for self-locking is,

$$x \leq \mu.a$$

Q 1) E) Explainthe terms slip and creep in belts. (05)

i) Slip of Belt:

Sometimes, the frictional grip becomes insufficient. This may cause some forward motion of the driver without carrying the belt with it. This may also cause some forward motion of the belt without carrying the driven pulley with it. This is called slip of the belt and is generally expressed as a percentage.

ii) Creep of Belt:

When the belt passes from the slack side to the tight side, a certain portion of the belt extends and it contracts again when the belt passes from the tight side to slack side. Due to these changes of length, there is a relative motion between the belt and the pulley surfaces. This relative motion is termed as creep. The total effect of creep is to reduce slightly the speed of the driven pulley or follower.

Q 1) F) State i) Wok enrgy principle ii) D'Alemberts principle. (05)

Solution:

i) Wok enrgy principle:

For a rigid body subjected to unbalanced forces and moments, the total work done in displacing the body from position $1 \rightarrow 2$ is equal to change in kinetic energy between two positions.

$$\therefore \mathrm{U}_{1\rightarrow 2} = \mathrm{KE}_2 - \mathrm{KE}_1$$

Where $\mathrm{U}_{1\rightarrow 2}$ = Total work done of the system in between initial and final position

KE_1 = Initial kinetic energy of body

KE_2 = Final kinetic energy of body

ii) D'Alemberts principle:

1) Consider a rigid body of mass m subjected to several external forces.

Let 'a' be the acceleration of mass center.

Let α be the angular acceleration of body.

2) Now if we apply an inertia force of magnitude $(\mathrm{m} \times \mathrm{a}_\mathrm{G})$ in opposite direction of acceleration of mass center and an inertia couple $\mathrm{I}\alpha$ in opposite direction of angular acceleration of the body is considered to be in dynamic equilibrium and the conditions of equilibrium can be applied, along with inertia force and couple, so under dynamic equilibrium we can use,

$$\sum \mathrm{F}_\mathrm{x} = 0, \qquad \sum \mathrm{F}_\mathrm{y} = 0, \qquad \text{and} \qquad \sum \mathrm{M}_\mathrm{G} = 0,$$

Q 2) A) In the toggle mechanism, as shown in Fig.1, the slider D is constrained to move on a horizontal path. The crank OA is rotating in the counter-clockwise direction at a speed of 180 r.p.m. The dimensions of various links are as follows : OA = 180 mm ; CB = 240 mm ; AB = 360

mm ; and BD = 540 mm. For the given configuration, find : 1. Velocity of slider D, 2. Angular velocity of links AB, CB and BD (12)

1. By Instantaneous centre method

2. By relative velocity meyhod

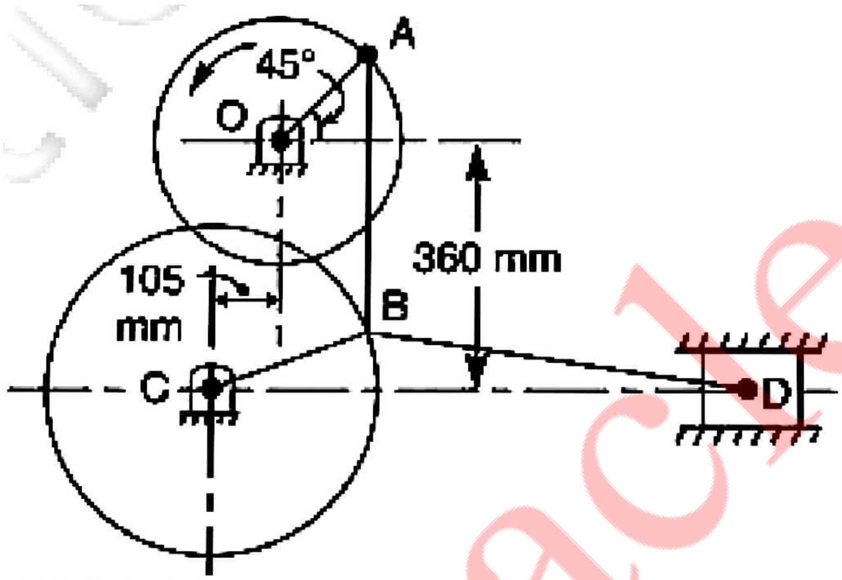


Figure No.1

Solution:

1. By Instantaneous centre method :

Speed of crank OA,

$$N_{AO} = 180 \text{ r.p.m. or } \omega_{AO} = 2\,\pi \times 180/60 = 18.85 \text{ rad/s}$$

Velocity of link AO is,

$$v_{AO} = v_A = \omega_{AO} \times OA = 18.85 \times 0.18 = 33.93 \text{ cm/s}$$

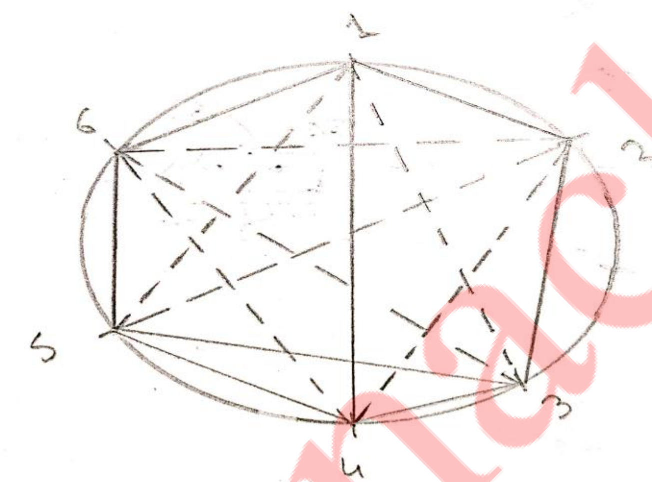
The No. of instantaneous centres are,

$$n = \frac{n(n-1)}{2} = \frac{6(6-1)}{2} = 15$$

All possible centres are

Instantaneous centres of mechanisms

Link	1	2	3	4	5	6
ICR	12	13	14	15	16	
	23	24	25	26		
	34	35	36			
	45	46				
	56					



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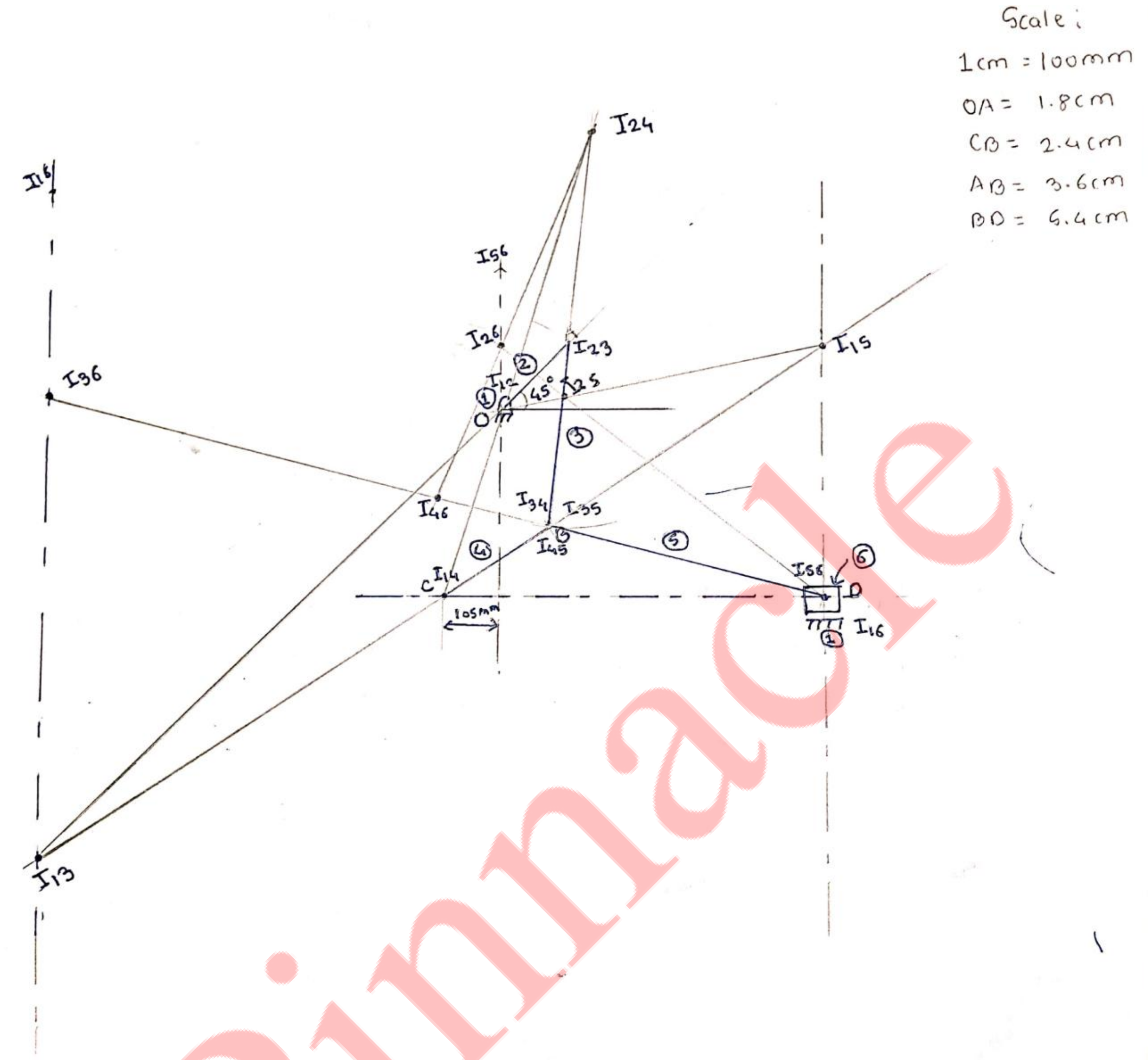


By measurement:

$I_{12}I_{23} = 1.8 \text{ cm}$

$I_{13}I_{23} = 14.1 \text{ cm}$

$I_{13}I_{35} = 11.6 \text{ cm}$

$I_{15}I_{35} = 5.8 \text{ cm}$

Velocity of links

$$v_{23} = \omega_2 \times (I_{12}I_{23}) = 18.85 \times 1.8 = 33.93 \text{ cm/s}$$

$$v_{23} = \omega_3 \times (I_{13}I_{23})$$

Velocity of link AB

$$\omega_3 = \frac{v_{23}}{I_{13}I_{23}} = \frac{33.93}{14.1} = 2.4\ \text{cm/s}$$

$$v_{35} = \omega_3 \times (I_{13}I_{35}) = 2.4 \times 11.6 = 27.84\ \text{cm/s}$$

Velocity of link BD

$$v_{35} = \omega_5 \times (I_{15}I_{35})$$

$$\omega_5 = \frac{v_{35}}{I_{13}I_{23}} = \frac{27.84}{6.8} = 4.09\ \text{cm/s}$$

Velocity of slider D

$$v_{I56} = \omega_5 \times (I_{15}I_{56}) = 4.09 \times 5 = 20.45\ \text{cm/s}$$

2. By relative velocity method :

$N_{AO} = 180\ \text{r.p.m. or}\ \omega_{AO} = 2\,\pi \times 180/60 = 18.85\ \text{rad/s}$

Since the crank length $OA = 180\ \text{mm} = 0.18\ \text{m}$, therefore velocity of A with respect to O or velocity of A (because O is a fixed point),

$$v_{AO} = v_A = \omega_{AO} \times OA = 18.85 \times 0.18 = 3.4\ \text{m/s}$$

… (Perpendicular to OA)

1. Velocity of slider D

First of all draw the space diagram, to some suitable scale, as shown in Fig. (a). Now the velocity diagram, as shown in Fig. (b), is drawn as discussed below :

1. Draw vector oa perpendicular to OA, to some suitable scale, to represent the velocity of A with respect to O or velocity of A (i.e. v_{AO} or v_A,) such that

$$\text{vector oa} = v_{AO} = v_A = 3.4\ \text{m/s}$$

2. Since point B moves with respect to A and also with respect to C, therefore draw vector ab perpendicular to AB to represent the velocity of B with respect to A i.e. v_{BA} and draw vector cb perpendicular to CB to represent the velocity of B with respect to C, i.e. v_{BC}. The vectors ab and cb intersect at b.

3. From point b, draw vector bd perpendicular to BD to represent the velocity of D with respect to B i.e. v_{DB}, and from point c draw vector cd parallel to the path of motion of the slider D (which is along CD) to represent the velocity of D, i.e. v_D. The vectors bd and cd intersect at d. By measurement, we find that velocity of the slider D,

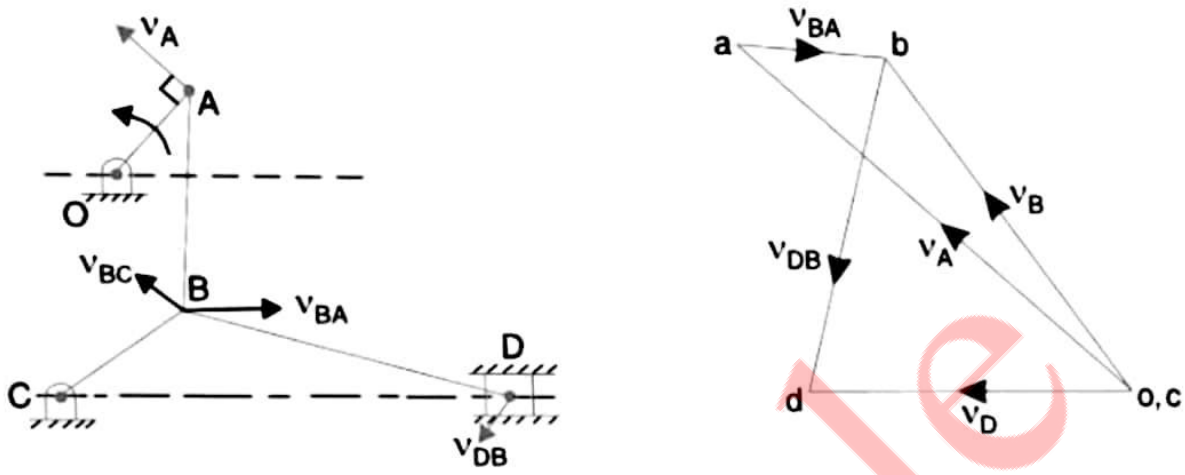
$$v_D = \text{vector cd} = 2.05\ \text{m/s}$$

2. Angular velocities of links AB, CB and BD

By measurement from velocity diagram, we find that

Velocity of B with respect to A,

$$v_{BA} = \text{vector ab} = 0.9 \text{ m/s}$$



(a) Space diagram (b) Velocity diagram

Velocity of B with respect to C,

$$v_{BC} = v_B = \text{vector cb} = 2.8 \text{ m/s}$$

and velocity of D with respect to B,

$$v_{DB} = \text{vector bd} = 2.4 \text{ m/s}$$

We know that $AB = 360 \text{ mm} = 0.36 \text{ m}$; $CB = 240 \text{ mm} = 0.24 \text{ m}$ and $BD = 540 \text{ mm} = 0.54 \text{ m}$.

∴ Angular velocity of the link AB,

$$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{0.9}{0.36} = 2.5 \text{ m/s(Anticlockwise about A)}$$

Similarly angular velocity of the link CB,

$$\omega_{CB} = \frac{v_{CB}}{CB} = \frac{2.8}{0.24} = 11.67 \text{ m/s(Anticlockwise about C)}$$

and angular velocity of the link BD,

$$\omega_{BD} = \frac{v_{BD}}{BD} = \frac{2.4}{0.54} = 4.44 \text{ m/(clockwise about B)}$$

Q 2) B) An open belt drive transmit power from a 300 mm diameter pulley running at 240 rpm to a pulley 450 mm diameter. Angle of lap on smaller pulley is 165°. The belt is on the point of slipping when 3KW is being transmitted. The coefficient of friction is 0.3. Determine effect on power transmission in following cases **(08)**

i) Initial tension in the belt increased by 10%

ii) Suitable dressing is given to the belt surface to increase the coefficient of friction by 10%. Assume that initial tension is kept same.

Solution:

Given: $d_1 = 300\ \text{mm} = 0.4\ \text{m}\ ;\ N_1 = 240\ \text{rpm}\ ;\ d_2 = 450\ \text{mm}\ ;\ \mu = 0.4\ ;$

$$P = 3\ \text{KW}\ ;\ \theta = 165° = 165 \times \frac{\pi}{180} = 2.88\ \text{rad}$$

Velocity of belt is

$$V = \frac{\pi \times d_1 \times N_1}{60} = \frac{\pi \times 0.3 \times 240}{60}$$

$$V = 3.78\ \text{m/s}$$

Limiting tension ratio is $\frac{T_1}{T_2} = e^{\mu.\theta}$

$$\frac{T_1}{T_2} = e^{0.3 \times 2.88} = 2.3724$$

$$T_1 = 2.3724\ T_2 \qquad \text{...(i)}$$

Power transmitted is

$$P = (T_1 - T_2)V$$

$$3 \times 10^3 = (2.3724\ T_2 - T_2) \times 3.78$$

$$T_2 = 578.29\ \text{N}$$

Put this value in equation (i)

$$T_1 = 2.3724 \times 578.29$$

$$= 1371.93\text{N}$$

i) If initial tension increased by 10%

Initial tension in belt is,

$$T_0 = \frac{T_1 + T_2}{2} = \frac{1371.93 + 578.29}{2}$$

$$= 975.11\ \text{N}$$

Increase in initial tension by 10%

$$T_0' = 1.10 T_0 = 1.10 \times 975.11$$

$$T_0' = 1072.62N$$

For increase in initial tension by 10% the corresponding increase in tension in tight and slack side is,

$$T_0' = \frac{T_1' + T_2'}{2}$$

$$1072.62 = \frac{T_1' + T_2'}{2}$$

$$2145.24 = T_1' + T_2'$$

$$2145.24 = 2.3724T_2' + T_2'$$

...(ratio of tension remains same)

$$T_2' = 636.12 \text{ N}$$

$$T_1' = 2.3724T_2' = 1509.13 \text{ N}$$

New power transmitted,

$$P' = (T_1' - T_2')V = (1509.13 - 636.12) \times 3.78$$

...(Velocity of belt remains same)

$$P' = 3.29 \text{ KW}$$

Percentage increase in power transmitted is,

$$= \frac{P' - p}{P} \times 100 = \frac{3.29 - 3}{3} \times 100$$

$$= 9.67\%$$

ii) Increase in coefficient of friction by 10% :

New coefficient of friction is,

$$\mu' = 1.10 \times \mu = 1.10 \times 0.3$$

$$= 0.33$$

New ratio of limiting tension is,

$$\frac{T_1}{T_2} = e^{\mu.\theta} = e^{0.33 \times 2.88} = 2.59$$

$$T_1 = 2.59\, T_2$$

Initial tension in belt is,

$$T_0 = \frac{T_1 + T_2}{2}$$

$$975.11 = \frac{T_1 + T_2}{2}$$

$$1950.22 = 2.59T_2 + T_2$$

$$T_2 = 543.23 \text{ N}$$

$$T_1 = 2.59 \times 543.23$$

$$T_1 = 1406.97 \text{ N}$$

New power transmitted is,

$$P' = (1406.97 - 543.23) \times 3.78$$

$$= 3.26 \text{ KW}$$

Percentage increase in power transmitted is,

$$= \frac{P' - p}{P} \times 100 = \frac{3.26 - 3}{3} \times 100$$

$$= 8.67\%$$

Q 3) A) Two 20° involute spur gear have a module of 10mm. The addendum is one module. The larger has 50 teeth and the pinion has 13 teeth. Does interference occur? If it occur , to what value the pressure angle be changed to eliminate interference. (10)

Solution:

$\varphi = 20°;\ T = 50: m = 10 \text{ mm}: t = 13;\ \ \text{Addendum} = 1\text{ m} = 10 \text{ mm}$

$R = \frac{mT}{2} = \frac{10 \times 50}{2} = 250 \text{ mm};$

$R_0 = 250 + 10 = 260 \text{ mm}$

$r = \frac{mt}{2} = \frac{10 \times 50}{2} = 65 \text{ mm};$

$R_{a\,max} = \sqrt{(R\cos\varphi)^2 + (R\sin\varphi + r\sin\varphi)^2}$

$= \sqrt{(250\cos 20°)^2 + (250\sin 20° + 65\sin 20°)^2}$

$= \sqrt{(250\cos 20°)^2 + (315\sin 20°)^2}$

$= 258.45 \text{ mm}$

The actual addendum radius R_a is more than the max. value $R_{a\,max}$ and therefore, interference occurs.

Maximum addendum radius can also be found using the relation

$$R_{a\,max} = R\sqrt{1 + \frac{1}{t}\left(\frac{t}{T} + 2\right)\sin^2\varphi}$$

$$= 250\sqrt{1 + \frac{13}{50}\left(\frac{13}{50} + 2\right)\sin^2\varphi} = 258.45\text{mm}$$

The new value of φ can be found by taking $R_{a\,max}$ equal to R_a

$$260 = \sqrt{(250\cos\varphi)^2 + (315\sin\varphi)^2}$$

$$(260)^2 = (250)^2\cos^2\varphi + (315)^2(1 - \cos^2\varphi)$$

$$= (250)^2\cos^2\varphi + (315)^2 - (315)^2\cos^2\varphi$$

$$\cos^2\varphi = \frac{(315)^2 - (260)^2}{(315)^2 - (250)^2} = 0.861$$

$$\cos\varphi = 0.928 \text{ or} \varphi = 21.88° \text{ or } 21°52'$$

Q 3) B) Differentiate between involute and cycloidal gear tooth profile. (04)

Solution:

Sr. No.	Involute tooth gear	Cycloidal tooth profile
1	The profile of involute gears is the single curvature.	The profile of cycloidal gears is double curvature i.e. epicycloidal and hypocycloid.
2	The pressure angle from start of engagement of teeth to the end of engagement remains constant, which results into smooth running.	The pressure angle varies from start of engagement to end of engagement, which results into less smooth running.
3	The center distance of involute gear can be varied within limits without changing the velocity ratio.	The center distance between cycloidal gears is to be kept constant to keep velocity ratio constant.
4	Manufacturing of involute gears is easy due to single curvature of tooth profile.	Manufacturing of cycloidal gear is difficult due to double curvature of tooth profile.

Q 3) C) In a crank and slotted lever quick return mechanism the distance between the fixed centres is 350 mm and length of driving crank is 150 mm. Find the inclination of slotted lever with vertical in the extreme position and ratio of time of cutting stroke to return stroke. (06)

Solution:

Given: $AC = 350 \text{ mm}$; $CB_1 = 150 \text{ mm}$

Inclination of the slotted bar with the vertical:

Let $\angle CAB_1$ = Inclination of the slotted bar with the vertical.

The extreme positions of the crank are shown in Fig. We know that

Let $\sin\angle CAB_1 = \sin\left(90° - \frac{\alpha}{2}\right)$

$$= \frac{B_1C}{AC} = \frac{150}{350} = 0.43$$

$$\therefore \angle CAB_1 = 90° - \frac{\alpha}{2}$$

$$= \sin^{-1}0.43 = 0.44 \times \frac{180}{\pi} = 25.21°$$

We know that

$90° - \alpha / 2 = 25.21°$

$\therefore \alpha / 2 = 90° - 25.21° = 64.79°$

or $\alpha = 2 \times 64.79° = 129.58°$

$$\therefore \frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{360° - 129.58°}{\alpha} = \frac{360° - 129.58°}{120°}$$

$$= 1.92$$

Q 4) A) A mechanism of crank and slotted lever quick return mechanism is shown in Figure 2. If the crank rotates counter clockwise at 120 r.p.m,. determine for the configuration shown, the velocity and acceleration of the ram D. Also determine the angular acceleration of the slotted lever. Crank = 150 mm ; Slotted arm, OC = 700 mm ; And link CD = 200 mm.

(14)

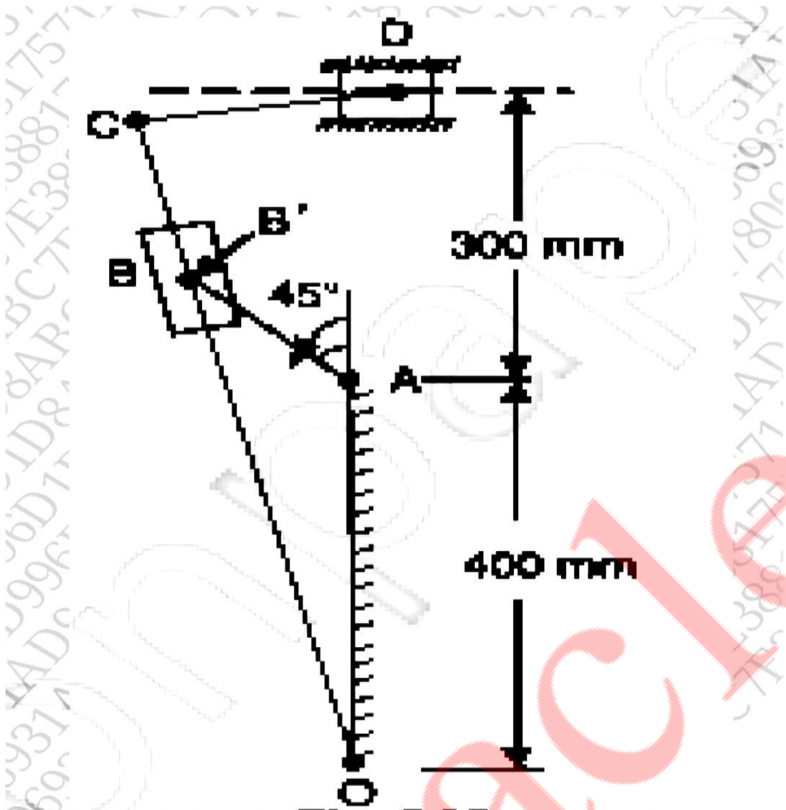


Figure No .2

Solution:

Given :$N_{BA} = 120$ r.p.m or $\omega_{BA} = 2\pi \times 120/60 = 12.57$rad/s;

$AB = 150$ mm $= 0.15$ m; $OC = 700$ mm $= 0.7$ m;

$CD = 200$ mm $= 0.2$ m

We know that velocity of B with respect to A,

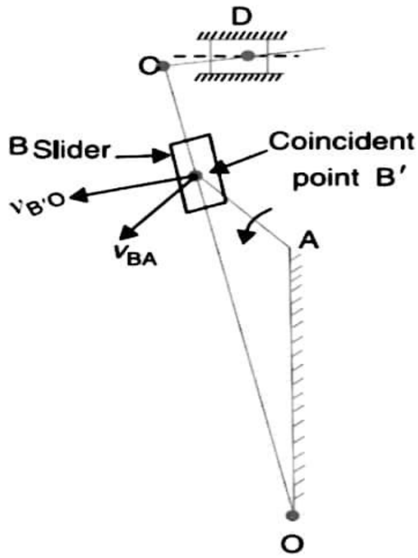
$$V_{BA} = \omega_{BA} \times AB$$

$= 12.57 \times 0.15 = 1.9$ m/s

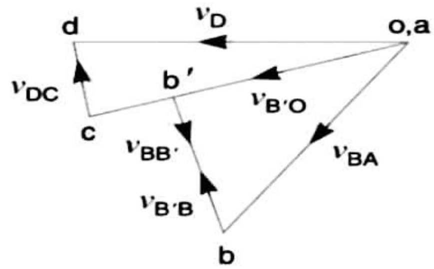
...(Perpendicular to AB)

Velocity of the ram D

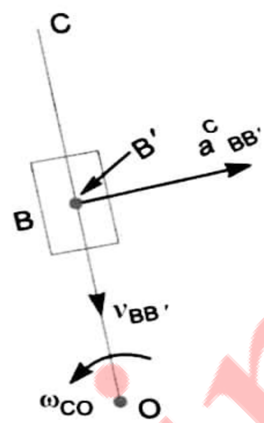
First of all draw the space diagram, to some suitable scale, as shown in Fig. (a). Now the velocity diagram, as shown in Fig. (b), is drawn as discussed below:



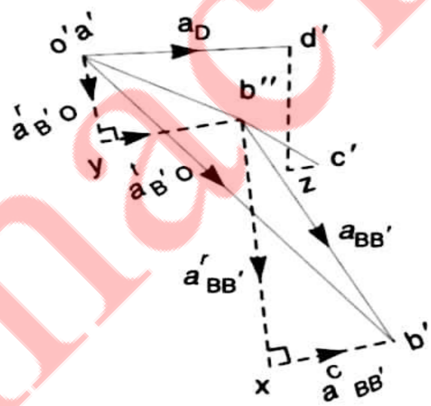
(a) Space diagram



(b) Velocity diagram



(c) Direction of coriolis component.



(d) Acceleration diagram

1. Since O and A are fixed points, therefore these points are marked as one point in velocity diagram. Now draw vector ab in a direction perpendicular to AB , to some suitable scale, to represent the velocity of slider B with respect to A i.e. V_{BA}, such that

vector ab $= V_{BA} = 1.9 \text{ m/s}$

2. From point o, draw vector ob' perpendicular to OB' to represent the velocity of coincident point B' (on the link OC) with respect to O i.e. $V_{B'o}$ and from point b draw vector bb' parallel to the path of motion of B' (which is along the link OC) to represent the velocity of coincident point B' with respect to the slider B i.e. $V_{B'B}$. The vectors ob' and bb' intersect at b'.

3. Since the point C lies on OB' produced, therefore, divide vector ob' at c in the same ratio as C divides OB' in the space diagram. In other words,

$$ob' / oc = OB' / OC$$

The vector oc represents the velocity of C with respect to O i.e. v_{CO}.

4. Now from point c, draw vector cd perpendicular to CD to represent the velocity of D with respect to C i.e. V_{DC} ,and from point o draw vector od parallel to the path of motion of D (which is along the horizontal) to represent the velocity of D i.e. V_D.The vectors cd and od intersect at d.

By measurement, we find that velocity of the ram D,

$$V_D = \text{vector od} = 2.15 \text{ m/s}$$

From velocity diagram, we also find that

Velocity of B with respect to B',

$$V_{b\prime b} = \text{vector b}'\text{b} = 1.05 \text{ m/s}$$

Velocity of D with respect to C,

$$V_{DC} = \text{vector cd} = 0.45 \text{ m/s}$$

Velocity of B' with respect to O

$$V_{B\prime O} = \text{vector ob}' = 1.55 \text{ m/s}$$

Velocity of C with respect to O,

$$V_{CO} = \text{vector oc} = 2.15 \text{ m/s}$$

∴ Angular velocity of the link OC or OB',

$$\omega_{CO} = \omega_{BO} = \frac{V_{CO}}{OC} = \frac{2.15}{0.7} = 3.07\text{rad/s(anticlockwise)}$$

Acceleration of the ram D

We know that radial component of the acceleration of B with respect to A,

$$a_{BA}^{r} = \omega_{BA}^{2} \times AB = (12.57)^2 \times 0.15 = 23.7 \text{ m/s}^2$$

Coriolis component of the acceleration of slider B with respect to the coincident point B',

$$a_{BA}^{r}{}' = 2\omega.v = 2\omega_{CO}.v_{BB} = 2 \times 3.07 \times 1.05 = 6.45 \text{ m/s}^2$$

Radial component of the acceleration of D with respect to C,

$$a_{BA}^{r} = \frac{V_{BA}^{2}}{CD} = \frac{(0.45)^2}{0.2} = 1.01 \text{ m/s}^2$$

Radial component of the acceleration of the coincident point B' with respect to O,

$$a^r_{B'O} = \frac{V^2_{B'O}}{BO} = \frac{(1.555)^2}{0.52} = 4.62\ m/s^2$$

...(By measurement B'O = 0.52 m)

Now the acceleration diagram, as shown in Fig. (d), is drawn

1. Since O and A are fixed points, therefore these points are marked as one point in the acceleration diagram. Draw vector a'b' parallel to AB, to some suitable scale, to represent the radial component of the acceleration of B with respect to A i.e. a^r_{BA} or a_B, such that

$$a'b' = a^r_{BA} = a_B = 23.7\ m/s^2$$

2. The acceleration of the slider B with respect to the coincident point B' has the following two components :

(i) Coriolis component of the acceleration of B with respect to B' i.e. a^c_{BB} , and

(ii) Radial component of the acceleration of B with respect to B' i.e. a^r_{BB}.

These two components are mutually perpendicular. Therefore from point b' draw vector b'x perpendicular to B'O i.e. in a direction as shown in Fig. (c) to represent$a^c_{BB} = 6.45\ m/s^2$

The direction of a^c_{BB} is obtained by rotating $V_{BB'}$ (represented by vector b'b in velocity diagram) through 90° in the same sense as that of link OC which rotates in the counter clockwise direction. Now from point x, draw vector xb'' perpendicular to vector b'x (or parallel to B'O) to represent a^r_{BB} whose magnitude is yet unknown.

3. The acceleration of the coincident point B' with respect to O has also the following two components:

(i) Radial component of the acceleration of coincident point B' with respect to O i.e. $a^r_{B'O}$, and

(ii) Tangential component of the acceleration of coincident point B' with respect to O, i.e. $a^t_{B'O}$

These two components are mutually perpendicular. Therefore from point o', draw vector o'y parallel to B'O to represent $a^r_{B'O} = 4.62\ m/s^2$ and from point y draw vector yb'' perpendicular to vector o'y to represent B $a^t_{B'O}$The vectors xb'' and yb'' intersect at b''. Join o'b''. The vector o'b'' represents the acceleration of B' with respect to O, i.e. $a_{B'O}$

4. Since the point C lies on OB' produced, therefore divide vector o'b'' at c' in the same ratio as C divides OB' in the space diagram. In other words,

$$o'b''/o'c' = OB'/OC$$

5. The acceleration of the ram D with respect to C has also the following two components:

(i) Radial component of the acceleration of D with respect to C i.e. a_{DC}^{r}, and

(ii) Tangential component of the acceleration of D with respect to C, i.e. a_{DC}^{t}.

The two components are mutually perpendicular. Therefore draw vector c'z parallel to CD to represent $a_{DC}^{r} = 1.01 \text{ m/s}^2$ and from z draw zd' perpendicular to vector zc' to represent a_{DC}^{t} at whose magnitude is yet unknown.

6. From point o', draw vector o'd' in the direction of motion of the ram D which is along the horizontal. The vectors zd' and o'd' intersect at d'. The vector o'd' represents the acceleration of ram D i.e. a_D

By measurement, we find that acceleration of the ram D,

$a_D = \text{vector } o'd' = 8.4 \text{ m/s}^2$

Angular acceleration of the slotted lever

By measurement from acceleration diagram, we find that tangential component of the coincident point B' with respect to O,

$$a_{B'O}^{t} = \text{vector } yb'' = 6.4 \text{ m/s}^2$$

We know that angular acceleration of the slotted lever,

$$= \frac{a_{B'O}^{t}}{OB'} = \frac{6.4}{0.52} = 12.3 \text{ rad/s}^2 (\text{anticlockwise})$$

Q 4) B) Draw a neat sketch of Tchebicheff mechanism and prove that the length of link must be in a ratio of 1:2:2.5. **(06)**

Solution:

1) In a figure the configuration of a chebychev mechanism is shown. The links are AB, CD, CB and AD.

$$AB = CD$$

P’ is the trace point. The dimensions are such that $BP = BP'$ .When link 4 rotates, P moves along an axis parallel to AD. This will be an approximate straight line.

2) Refer Fig. a. It is a four-bar mechanism with crossed links PB and QA of equal length. Tracing point R is situated at the mid-point of AB and usually the prportions of the extreme position on the left and Q, A and B fall in a vertical line on the extreme position on the left and Q, A and B fall in a vertical line on the extreme position on the right. These position are shown in Fig. a. by the configuration QA_2B_2 and PBA_1Q. Thus condition leads to fixed ratio of lengths of AB: PQ: PB. The mechanism is drawn in four different configurations and the locus of point R is a

straight line parallel to PQ. The instantaneous centre of link 3 i.e. O_{31} always lies vertically bwlow R at the intersection of the two links PB and QA and, therefore, motion of point R is horizontal or perpendicular to line joining P to O_{31}.

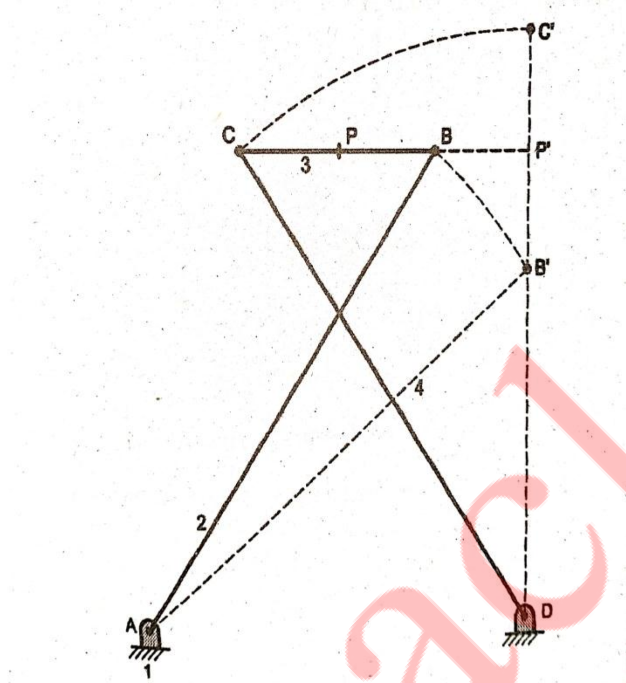


FIG : Tchebicheff mechanism

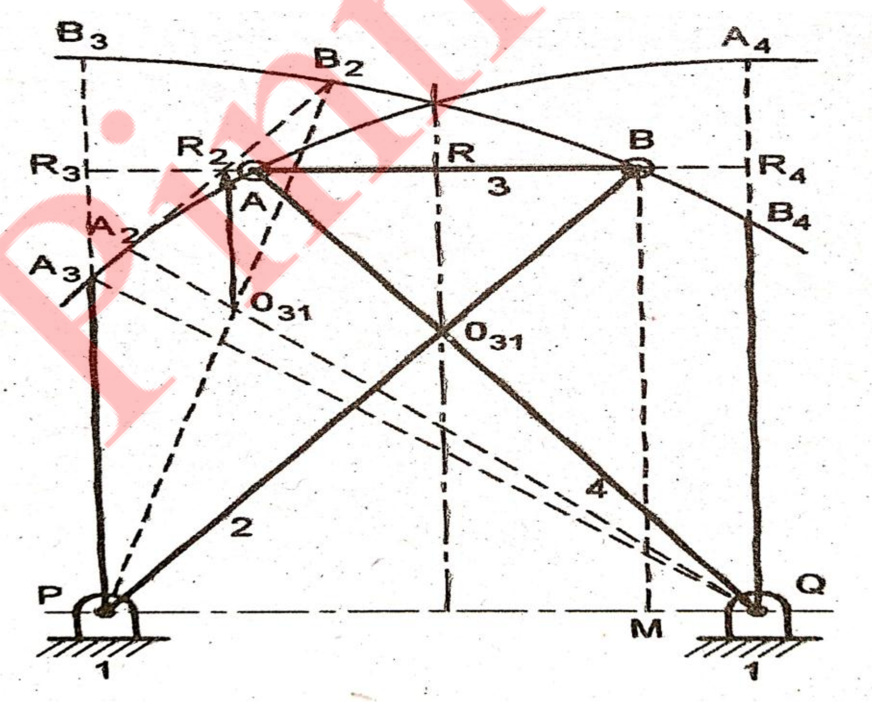


FIG : (a)

Calculate the lengths of the line links with above mentioned limits for extreme positions

Let $AB = 1$ unit length

$PB = QA = a$ units of length

$PQ = b$ units of length

From triangle PA_3Q

$$PA_3^2 = A_3Q^2 - PQ^2$$

$$(PB_3 - A_3B_5)^2 = A_3Q^2 - PQ^2$$

$$(a-1)^2 = a^2 - b^2$$

$$a^2 - 2a + 1 = a^2 - b^2$$

$$b^2 = 2a - 1$$

$$a = \frac{b^2+1}{2} \qquad \text{...(i)}$$

Let BM perpendicular from point B to PQ.

Therefore, $BP^2 = BM^2 + BM^2$

$$a^2 = \left(a - \frac{1}{2}\right)^2 + \left(b - \frac{1}{2}\right)^2$$

$$= a^2 - a + 1/4 + b^2 - b + 1/4$$

$$a + b - b^2 = \frac{1}{2} \qquad \text{...(ii)}$$

From Equation (i) and (ii), we get

$$\frac{b^2+1}{2} + b - b^2 = \frac{1}{2}$$

$$-\frac{b^2}{2} + b = 0$$

Therefore, $b = 2$

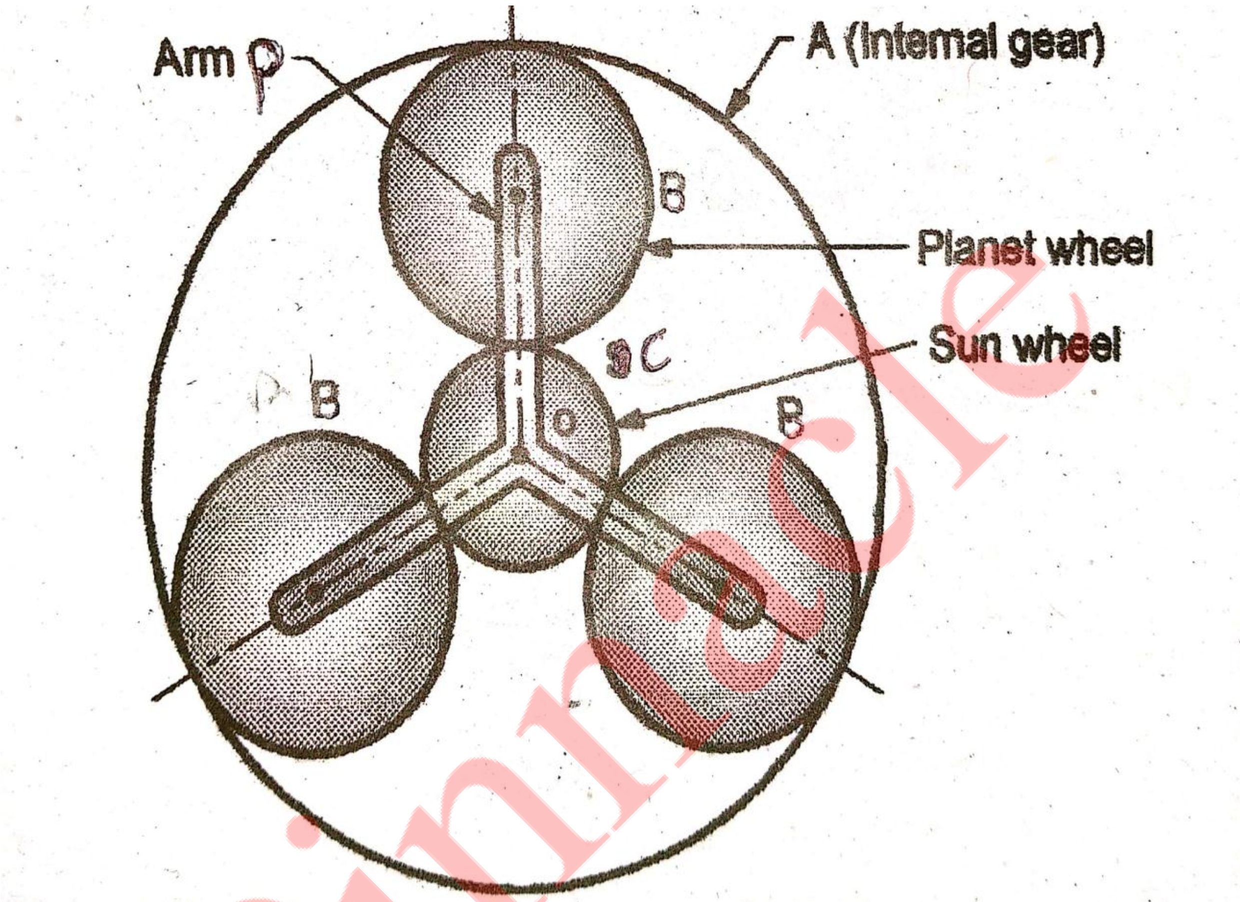
$$a = \frac{b^2+1}{2} = \frac{4+1}{2} = 2.5$$

Thus $AB : PQ : PB = 1 : 2 : 2.5.$ ...proved

Q 5) A) In an epicyclic gear train an annular wheel A having 54 teeth meshes with a planet wheel B which gears with a sun wheel C, the wheel A and C being co-axial. The wheel B is carried on a pin fixed on one end of arm P which rotates at 100 rpm about the axis of the

wheel A and C. If the wheel A makes 20 rpm in clockwise sense and the arm rotates at 100 rpm in anti clockwise direction and C has 24 teeth, Sketch the arrangement and determine rpm and sense of rotation of wheel C. **(08)**

Solution:



Given: $t_A = 54$; $N_P = -100$ r.p.m. ; $t_C = 24$; $N_A = 20$;

Referring to fig. we have

Since number of teeth are proportional to their pitch circle diameters, it follows that

$$t_C + 2t_B = t_A$$

$$t_B = \frac{t_A - t_C}{2} = \frac{54-24}{2}$$

$$t_B = 15 \text{ teeth}$$

Steps	Operation	Revolutions of elements (N)			
		Arm P	Gear C ($t_C = 24$)	Gear B ($t_B = 24$)	Internal gear ($t_A = 24$)
1.	Fix the spider of arm – P and given +1 revolution to 'S'	0	$+1$	$-\frac{t_C}{t_B}$	$-\frac{t_C}{t_A}$
2.	Multiply 'm'	0	m	$-m.\frac{t_C}{t_B}$	$-m.\frac{t_C}{t_A}$
3.	Add 'n' revolutions to all elements	n	n	n	n
4.	Total motion	n	$m+n$	$-m.\frac{t_C}{t_B}+n$	$-m.\frac{t_C}{t_A}+n$

$$N_A = 20 = -m.\frac{t_C}{t_A} + n$$

$$20 = -m.\frac{24}{54} + n \quad \text{...(i)}$$

$$N_P = n = -100$$

Adding value of 'n' in equation (i)

$$20 = -m.\frac{24}{54} - 100$$

$$20 = -0.44m - 100$$

$$m = -272.72$$

Since, from table

$$N_C = m + n = -272.72 - 100 = -372.72 \text{ r.p.m.}$$

$$N_C = 372.72 \text{ r.p.m. (anticlockwise)}$$

Q 5) B) A cord wrapped around a solid cylinder of radius 'r' and mass 'm'. The cylinder is released from rest. Determine the velocity of it's centre of mass after it has moved down a distance 'h'. (06)

Solution:

Consider dynamic equilibrium of cylinder, by applying D'Alembert's force $m \times a$ and $I\alpha$ in opposite direction of rotation.

Apply, $\sum F_y = 0$, $\quad T + ma - mg = 0$

$$T = mg - ma$$

$$= m(g - a)$$

$\sum M_G = 0$, $\quad -T.r + I_G.\alpha = 0$

$$T.r = I_G.\alpha$$

But $\alpha = a/r$ and $I_G = \frac{mr^2}{2}$

$$T.r = \frac{mr^2}{2} \times \frac{a}{r}$$

$$T = \frac{ma}{2} \quad \text{...(2)}$$

Eqating equations (1) and (2)

$$m(g - a) = \frac{ma}{2}$$

$$g - a = a/2$$

$$g = \frac{3a}{2}$$

$$a = \frac{2g}{3}$$

And the velocity will be,

$$v = \frac{2g}{3} \times t$$

Q 5) C) Two shafts are connected by Hooke's joint. The driving shafts rotates at a uniform speed of 1000 rpm. The angle between the shafts is 20°. Calculate the maximum and minimum speed of driven shafts, when acceleration of the driven shaft is maximum. (06)

Solution:

Given: Driving shaft speed $N_1 = 1000$ rpm

Angle between shafts $\alpha = 20°$

i) Maximum speed of driven shaft,

$$(N_2)_{max} = \frac{N_1}{\cos\alpha} = \frac{1000}{\cos 20°}$$

$$= 2450.48 \text{ rpm}$$

ii) Minimum speed of the driven shaft

$$(N_2)_{min} = N_1 . \cos\alpha = 1000 \times \cos 20°$$

$$= 408.08 \text{ rpm}$$

Q 6) A) The center to center distance between thw two sprockets of a chain drive is 600 mm. The chain drive is used to reduce the speed from 180 rpm to 90 rpm on the driving sprocket has 18 teeth and a pitch circle diameter of 480 mm. Determine the i) Number of teeth on driven sprocket ii) Pitch and length of chain. (08)

Solution:

i) $$\frac{N_2}{N_1} = \frac{T_1}{T_2}$$

or $$T_2 = T_1 \frac{N_2}{N_1} = 18 \times \frac{180}{90} = 36$$

ii) $$p = 2r \sin\frac{180°}{T} = 2 \times 0.24 \times \sin\frac{180°}{36}$$

$$= 0.0418 \text{ or } 41.8 \text{ mm}$$

$$k = C/p = 0.600/0.0418 = 14.342$$

$$L = p\left[\frac{T+t}{2} + \frac{\left(\text{cosec}\frac{180°}{T} - \text{cosec}\frac{180°}{t}\right)^2}{4k} + 2k\right]$$

$$= 0.0418\left[\frac{36+18}{2} + \frac{\left(\text{cosec}\frac{180°}{36} - \text{cosec}\frac{180°}{18}\right)^2}{4\times14.342} + 2 \times 14.342\right]$$

$$= 0.0418 \times (27 + 0.569 + 28.684)$$

$$= 2.351 \text{ m}$$

Q 6) B) A cam is rotating at 200 rpm operate a reciprocating roller follower of radius 2.5 cm. The least radius of cam is 30 mm, stroke of follower is 5 cm. Ascent takes place by uniform acceleration and deceleration and descent by simple harmonic motion. Ascent take place by 70° and descent during 50° of cam rotation. Dwell between ascent and descent 60°. Sketch displacement, velocity, acceleration and jerk diagram. (12)

Solution:

Cam speed $N = 200$ rpm

$$\therefore \omega = \frac{2\pi N}{60} = 20.94 \text{ rad/sec}$$

Out stroke, $S = 5 \text{ cm} = 50 \text{ mm}$

Angle of out stroke, $\theta_0 = 70°$

Angle of out return, $\theta_r = 50°$

Angle of out dwell, $\theta_d = 60°$

Follower moves with UARM for out stroke:

Displacement during outstroke i.e. 70° cam rotation

$y_{o°} = 70°$

$y_{35°} = \frac{s}{2} = \frac{50}{2} = 25 \text{ mm}$

$y_{0°} = S = 70°$

Velocity will be,

$V_0 = 0°$

$y_{35°} = \frac{2\omega s}{\theta_o} = \frac{2 \times 50 \times 20.94}{\left(\frac{70}{180}\pi\right)} = 1713.96 \text{ mm/sec}$

$= 1.713 \text{ m/sec}$

$y_{70°} = 0°$

Acceleration is constant,

$f_o = \frac{\pm 4 \text{ s } \omega^2}{(\theta_o)^2} = \frac{\pm 4 \times 50 \times \ (20.94)^2}{\left(\frac{70}{180}\pi\right)^2}$

$= \pm 58.75 \text{ m/sec}$

Jerk for uniform acceleration will be ∞ for all position of cam angle

$$P_o = \infty$$

Follower moves with stern for return stroke:

Displacement using return stroke i.e. 120° of cam angle

$y_{75°+60°+0°} = y_{135°} = s = 50$

$y_{75°+60°+25°} = y_{160°} = \frac{s}{2} = \frac{50}{2} = 25 \text{ mm}$

$y_{75°+60°+0°} = y_{185°} = 0 \text{ mm}$

Velocity will be,

$$V_{135^\circ} = 0$$

$$V_{160^\circ} = \frac{-\pi}{\theta_r}\frac{s}{2}\omega^2$$

$$= \frac{250}{\left(\frac{70}{180}\pi\right)} \times \frac{50}{2} \times 20.94$$

$$= 1884.6 \text{ mm/sec} = 1.88 \text{ m/sec}$$

$$V_{185^\circ} = 0$$

Acceleration will be,

$$f_{135^\circ} = \frac{-\pi^2 s}{(\theta_r)^2 2}\omega^2$$

$$= \frac{-\pi^2}{\left(\frac{50}{180}\pi\right)^2} \times \frac{50}{2} \times (20.94)^2$$

$$= -142.06 \times 10^3 \text{ mm/sec}^2$$

$$= -142.06 \ \text{ m/sec}^2$$

$$f_{160^\circ} = 0$$

$$f_{185^\circ} = \frac{\pm\pi^2 s}{(\theta_r)^2 2}\omega^2 = \frac{\pm\pi^2}{\left(\frac{50}{180}\pi\right)^2} \times \frac{50}{2} \times (20.94)^2$$

$$= \pm 142.06 \times 10^3 \text{ mm/sec}^2$$

$$= \pm 142.06 \ \text{ m/sec}^2$$

Jerk will be,

$$P_{135^\circ} = 0$$

$$P_{135^\circ} = \frac{\pi^3 s}{(\theta_r)^3 \ 2}\omega^2 = \frac{\pi^3}{\left(\frac{50}{180}\pi\right)^3} \times \frac{50}{2} \times (20.94)^3$$

$$= 10.70 \times 10^6 \text{ mm/sec}^2$$

$$= \pm 10.70 \times 10^3 \ \text{ m/sec}^2$$

$$P_{185^\circ} = 0$$

Q.P Code: 71865